# LEAST COST FORMULATIONS, LTD. 

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## TECHNICAL REPORT

NUMBER: TR308
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TITLE: $\quad$ Sampling plans to verify the proportion of an event exceeds or falls below a specified value.

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#### Abstract

Tables of sampling plans are provided that ensure that the 1-sided modified Wilson $95 \%$ confidence interval for a proportion falls above or below a specified value. Exhibit 1 shows the low range, where a proportion must fall below a specified value, and Exhibit 2 shows a high range, where a proportion must fall above a specified value. For example, in Exhibit 1, if the proportion of an event must fall below $5 \%$ with $95 \%$ 1-sided confidence, the sample size of 60 with no occurrences of the event will suffice. Similarly for a sample size of 90 with one or less occurrences of the event. For this latter sampling plan, if one occurrence is actually observed, the $95 \% 2$-sided confidence interval on the proportion will be [ $0.0 \%, 6.0 \%$ ], and the "average outgoing quality level" of the plan will be $3.0 \%$ (i.e., the process must be at $3 \%$ or less to pass the plan on the average). The plans of Exhibit 2 are interpreted in a similar way. For example, if the proportion of an event must fall above $95 \%$ with $95 \%$ 1-sided confidence, the sample size of 60 with all being occurrences of the event will suffice. Similarly for a sample size of 90 with 89 or more occurrences.


KEYWORDS: 1) PROPORTION 2) WILSON 3) SAMPLE SIZE

REL.DOC.: TR258
REVISED:

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ASSUME:
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1. Binary outcome (occur / not occur).
2. Constant probability rho of event occurring.
3. Independent trials (e.g., simple random sample).
4. Fixed number of trials N .

INFERENCE: $95 \%$ confidence interval lies entirely at or BELOW specified maximum rho.
DESIRED: Sample size N needed.
NOTES: 1. Based on modified Wilson score 1-sided confidence interval.
2. $A O Q L=$ Average Outgoing Quality Level

| Maximum <br> Probability rho | Sample Size N | Maximum Number Events x | Minimum Number Non-events y | 1-sided Upper Confidence Limit on rho | Expected Lower Confidence Limit on rho | Expected Upper Confidence Limit on rho | $\begin{gathered} \text { Effective } \\ \text { AOQL } \\ \text { rho } \\ \hline \end{gathered}$ |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 50\% | 3 | 0 | 3 | 47.4\% | 0.0\% | 56.1\% | 28.1\% |
| 50\% | 10 | 2 | 8 | 45.9\% | 5.7\% | 51.0\% | 28.3\% |
| 50\% | 20 | 6 | 14 | 48.4\% | 14.5\% | 51.9\% | 33.2\% |
| 50\% | 40 | 14 | 26 | 48.0\% | 22.1\% | 50.5\% | 36.3\% |
| 50\% | 80 | 32 | 48 | 49.2\% | 30.0\% | 51.0\% | 40.5\% |
| 45\% | 2 | 0 | 2 | 57.5\% | 0.0\% | 65.8\% | 32.9\% |
| 45\% | 10 | 1 | 9 | 34.8\% | 0.0\% | 40.4\% | 20.2\% |
| 45\% | 20 | 5 | 15 | 43.2\% | 11.2\% | 46.9\% | 29.0\% |
| 45\% | 40 | 12 | 28 | 42.9\% | 18.1\% | 45.4\% | 31.8\% |
| 45\% | 80 | 28 | 52 | 44.1\% | 25.5\% | 45.9\% | 35.7\% |
| 40\% | 5 | 0 | 5 | 35.1\% | 0.0\% | 43.4\% | 21.7\% |
| 40\% | 10 | 1 | 9 | 34.8\% | 0.0\% | 40.4\% | 20.2\% |
| 40\% | 20 | 4 | 16 | 37.8\% | 8.1\% | 41.6\% | 24.8\% |
| 40\% | 40 | 10 | 30 | 37.6\% | 14.2\% | 40.2\% | 27.2\% |
| 40\% | 80 | 24 | 56 | 39.0\% | 21.1\% | 40.8\% | 30.9\% |
| 35\% | 6 | 0 | 6 | 31.1\% | 0.0\% | 39.0\% | 19.5\% |
| 35\% | 10 | 1 | 9 | 34.8\% | 0.0\% | 40.4\% | 20.2\% |
| 35\% | 20 | 3 | 17 | 32.2\% | 5.2\% | 36.0\% | 20.6\% |
| 35\% | 40 | 9 | 31 | 34.9\% | 12.3\% | 37.5\% | 24.9\% |
| 35\% | 80 | 21 | 59 | 35.0\% | 17.9\% | 36.8\% | 27.3\% |
| 30\% | 7 | 0 | 7 | 27.9\% | 0.0\% | 35.4\% | 17.7\% |
| 30\% | 10 | 0 | 10 | 21.3\% | 0.0\% | 27.8\% | 13.9\% |
| 30\% | 20 | 2 | 18 | 26.2\% | 2.8\% | 30.1\% | 16.4\% |
| 30\% | 40 | 7 | 33 | 29.3\% | 8.7\% | 31.9\% | 20.3\% |
| 30\% | 80 | 17 | 63 | 29.6\% | 13.7\% | 31.4\% | 22.6\% |
| 25\% | 9 | 0 | 9 | 23.1\% | 0.0\% | 29.9\% | 15.0\% |
| 25\% | 10 | 0 | 10 | 21.3\% | 0.0\% | 27.8\% | 13.9\% |
| 25\% | 20 | 1 | 19 | 19.6\% | 0.0\% | 23.6\% | 11.8\% |
| 25\% | 40 | 5 | 35 | 23.5\% | 5.5\% | 26.1\% | 15.8\% |
| 25\% | 80 | 13 | 67 | 24.1\% | 9.7\% | 25.8\% | 17.8\% |
| 20\% | 11 | 0 | 11 | 19.7\% | 0.0\% | 25.9\% | 12.9\% |
| 20\% | 20 | 1 | 19 | 19.6\% | 0.0\% | 23.6\% | 11.8\% |
| 20\% | 40 | 3 | 37 | 17.3\% | 2.6\% | 19.9\% | 11.2\% |
| 20\% | 80 | 10 | 70 | 19.8\% | 6.9\% | 21.5\% | 14.2\% |
| 15\% | 20 | 0 | 20 | 11.9\% | 0.0\% | 16.1\% | 8.1\% |
| 15\% | 40 | 2 | 38 | 14.0\% | 1.4\% | 16.5\% | 8.9\% |
| 15\% | 80 | 6 | 74 | 13.9\% | 3.5\% | 15.4\% | 9.4\% |
| 10\% | 40 | 0 | 40 | 6.3\% | 0.0\% | 8.8\% | 4.4\% |
| 10\% | 60 | 2 | 58 | 9.6\% | 0.9\% | 11.4\% | 6.1\% |
| 10\% | 80 | 3 | 77 | 9.0\% | 1.3\% | 10.5\% | 5.9\% |
| 5\% | 60 | 0 | 60 | 4.3\% | 0.0\% | 6.0\% | 3.0\% |
| 5\% | 80 | 0 | 80 | 3.3\% | 0.0\% | 4.6\% | 2.3\% |
| 5\% | 90 | 1 | 89 | 4.8\% | 0.0\% | 6.0\% | 3.0\% |
| 5\% | 96 | 1 | 95 | 4.5\% | 0.0\% | 5.7\% | 2.8\% |
| 2\% | 130 | 0 | 130 | 2.0\% | 0.0\% | 2.9\% | 1.4\% |
| 2\% | 240 | 1 | 239 | 1.8\% | 0.0\% | 2.3\% | 1.2\% |
| 1\% | 280 | 0 | 280 | 1.0\% | 0.0\% | 1.4\% | 0.7\% |
| 1\% | 480 | 1 | 479 | 0.9\% | 0.0\% | 1.2\% | 0.6\% |


| ASSUME: | 1. Binary outcome (occur / not occur). <br> 2. Constant probability rho of event occurring. <br> 3. Independent trials (e.g., simple random sample). <br> 4. Fixed number of trials N . |  |  |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| INFERENCE: DESIRED: NOTES: | \% confidence mple size N n Based on mod AOQL = Avera | terval lies enti ded. ed Wilson sco e Outgoing | ly at or ABOVE <br> 1 -sided confid lity Level | specified minim nce interval. | um rho. |  |  |
| Minimum <br> Probability rho | Sample Size N | Minimum <br> Number <br> Events <br> x | Maximum <br> Number Non-events y | 1-sided <br> Lower <br> Confidence <br> Limit on rho | Expected Lower Confidence Limit on rho | Expected Upper Confidence Limit on rho | $\begin{gathered} \text { Effective } \\ \text { AOQL } \\ \text { rho } \\ \hline \end{gathered}$ |
| 50\% | 3 | 3 | 0 | 52.6\% | 43.8\% | 100.0\% | 71.9\% |
| 50\% | 10 | 8 | 2 | 54.1\% | 49.0\% | 94.3\% | 71.7\% |
| 50\% | 20 | 14 | 6 | 51.6\% | 48.1\% | 85.5\% | 66.8\% |
| 50\% | 40 | 26 | 14 | 52.0\% | 49.5\% | 77.9\% | 63.7\% |
| 50\% | 80 | 48 | 32 | 50.8\% | 49.0\% | 70.0\% | 59.5\% |
| 55\% | 4 | 4 | 0 | 59.7\% | 51.0\% | 100.0\% | 75.5\% |
| 55\% | 10 | 9 | 1 | 65.2\% | 59.6\% | 100.0\% | 79.8\% |
| 55\% | 20 | 15 | 5 | 56.8\% | 53.1\% | 88.8\% | 71.0\% |
| 55\% | 40 | 28 | 12 | 57.1\% | 54.6\% | 81.9\% | 68.2\% |
| 55\% | 80 | 52 | 28 | 55.9\% | 54.1\% | 74.5\% | 64.3\% |
| 60\% | 5 | 5 | 0 | 64.9\% | 56.5\% | 100.0\% | 78.3\% |
| 60\% | 10 | 9 | 1 | 65.2\% | 59.6\% | 100.0\% | 79.8\% |
| 60\% | 20 | 16 | 4 | 62.2\% | 58.4\% | 91.9\% | 75.2\% |
| 60\% | 40 | 30 | 10 | 62.4\% | 59.8\% | 85.8\% | 72.8\% |
| 60\% | 80 | 56 | 24 | 61.0\% | 59.2\% | 78.9\% | 69.1\% |
| 65\% | 6 | 6 | 0 | 68.9\% | 61.0\% | 100.0\% | 80.5\% |
| 65\% | 10 | 9 | 1 | 65.2\% | 59.6\% | 100.0\% | 79.8\% |
| 65\% | 20 | 17 | 3 | 67.8\% | 64.0\% | 94.8\% | 79.4\% |
| 65\% | 40 | 31 | 9 | 65.1\% | 62.5\% | 87.7\% | 75.1\% |
| 65\% | 80 | 59 | 21 | 65.0\% | 63.2\% | 82.1\% | 72.7\% |
| 70\% | 7 | 7 | 0 | 72.1\% | 64.6\% | 100.0\% | 82.3\% |
| 70\% | 10 | 10 | 0 | 78.7\% | 72.2\% | 100.0\% | 86.1\% |
| 70\% | 20 | 18 | 2 | 73.8\% | 69.9\% | 97.2\% | 83.6\% |
| 70\% | 40 | 33 | 7 | 70.7\% | 68.0\% | 91.3\% | 79.7\% |
| 70\% | 80 | 63 | 17 | 70.4\% | 68.6\% | 86.3\% | 77.4\% |
| 75\% | 9 | 9 | 0 | 76.9\% | 70.1\% | 100.0\% | 85.0\% |
| 75\% | 10 | 10 | 0 | 78.7\% | 72.2\% | 100.0\% | 86.1\% |
| 75\% | 20 | 19 | 1 | 80.4\% | 76.4\% | 100.0\% | 88.2\% |
| 75\% | 40 | 35 | 5 | 76.5\% | 73.9\% | 94.5\% | 84.2\% |
| 75\% | 80 | 67 | 13 | 75.9\% | 74.2\% | 90.3\% | 82.2\% |
| 80\% | 11 | 11 | 0 | 80.3\% | 74.1\% | 100.0\% | 87.1\% |
| 80\% | 20 | 19 | 1 | 80.4\% | 76.4\% | 100.0\% | 88.2\% |
| 80\% | 40 | 37 | 3 | 82.7\% | 80.1\% | 97.4\% | 88.8\% |
| 80\% | 80 | 70 | 10 | 80.2\% | 78.5\% | 93.1\% | 85.8\% |
| 85\% | 20 | 20 | 0 | 88.1\% | 83.9\% | 100.0\% | 91.9\% |
| 85\% | 40 | 38 | 2 | 86.0\% | 83.5\% | 98.6\% | 91.1\% |
| 85\% | 80 | 74 | 6 | 86.1\% | 84.6\% | 96.5\% | 90.6\% |
| 90\% | 40 | 40 | 0 | 93.7\% | 91.2\% | 100.0\% | 95.6\% |
| 90\% | 60 | 58 | 2 | 90.4\% | 88.6\% | 99.1\% | 93.9\% |
| 90\% | 80 | 77 | 3 | 91.0\% | 89.5\% | 98.7\% | 94.1\% |
| 95\% | 60 | 60 | 0 | 95.7\% | 94.0\% | 100.0\% | 97.0\% |
| 95\% | 80 | 80 | 0 | 96.7\% | 95.4\% | 100.0\% | 97.7\% |
| 95\% | 90 | 89 | 1 | 95.2\% | 94.0\% | 100.0\% | 97.0\% |
| 95\% | 96 | 95 | 1 | 95.5\% | 94.3\% | 100.0\% | 97.2\% |
| 98\% | 130 | 130 | 0 | 98.0\% | 97.1\% | 100.0\% | 98.6\% |
| 98\% | 240 | 239 | 1 | 98.2\% | 97.7\% | 100.0\% | 98.8\% |
| 99\% | 280 | 280 | 0 | 99.0\% | 98.6\% | 100.0\% | 99.3\% |
| 99\% | 480 | 479 | 1 | 99.1\% | 98.8\% | 100.0\% | 99.4\% |

